# **Module Leader**

Dr T Butlin [1]

#### Lecturers

Prof D Cebon, Dr T Butlin

#### Lab Leader

Dr T Butlin [1]

# **Timing and Structure**

Lent term. Vibration of Continuous Systems: 1 lecture/week, weeks 1-8 Lent term (Dr T Butlin), Vibration of Lumped Systems: Rayleigh's method, 1 lecture/week, weeks 1-8 Lent term (Prof D Cebon). 16 lectures.

# **Prerequisites**

3C5 useful (there is one particular result from the Lagrange section of 3C5 which will be quoted without proof)

# **Aims**

The aims of the course are to:

- Introduce the central ideas and tools for the analysis of small (linear) vibration in mechanical systems.
- Introduce simple continuous systems which may be combined as components of larger systems.
- Introduce the general approach to lumped systems via mass and stiffness matrices, and the resulting properties of vibration modes and their frequencies.

# **Objectives**

As specific objectives, by the end of the course students should be able to:

- Derive the partial differential equations governing the forced or free motion of uniform one-dimensional systems.
- Use these equations and appropriate boundary conditions to obtain vibration modes and natural frequencies.
- Analyse continuous systems using modal methods.
- Compute impulse and harmonic response by modal and direct methods.
- Be able to derive the dispersion relation for wave propagation in 1D structures.
- Understand that vibration can be expressed in terms of wave propagation or superposition of modes.
- Calculate the response of a coupled system from a knowledge of the responses of the separate subsystems.
- Apply Rayleigh's method to continuous systems.
- Take advantage of the link between Lagrange's equations and small vibration.
- Explain the concept of a vibration mode, and be able to find the modes and their natural frequencies by an

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- eigenvector/eigenvaluecalculation.
- Understand orthogonality of modes, modal damping, modal density and modal overlap factor.
- Express the frequency response functions or the impulse response functions of a system in terms of the normal modes, and be familiar with the concepts of resonances and antiresonances.
- Recognise and apply the reciprocal theorem for responses.
- Use the stationary property of normal mode frequencies to estimate frequencies given assumed mode shapes, using minimisation with respect to any free parameters.

# Content

This course aims to present a systematic approach to the study of small vibration of engineering components and structures. The course picks up where Part IA Linear Systems and Vibration left off. Concepts which were barely discussed (e.g. reciprocity and the orthogonality of vibration modes) are important for building up qualitative insights into vibration behaviour. Alongside the mathematical tools for quantitative analysis the course offers vital ingredients for an engineer's education.

### **Vibration of Continuous Systems (8L)**

- Vibration of strings; axial and transverse vibration of beams, torsional vibration of circular shafts; 1D acoustic vibration in a duct;
- Modal analysis of simple systems;
- Electrical transmission line analogy of 1D mechanical wave propagation;
- D'Alembert's solution;
- Dispersion relation for travelling waves;
- Response to impulse and harmonic excitation;
- Transfer functions and the meaning of poles and zeros;
- · Coupling of systems;
- Rayleigh's method for continuous systems.

# Vibration of Lumped Systems (8L)

- Application of Lagrange's equations to small vibrations; undamped vibration of systems with N degrees of freedom;
- · Matrix methods and modal analysis;
- Response functions in frequency and time domains; properties of frequency-response functions; reciprocal theorems;
- Modal damping and modal overlap;
- Rayleigh's method for discrete systems.

### Coursework

A data-logging and FFT analysis system is used to measure the frequency response of a vibrating system by three different methods, to compare their merits and disadvantages.

#### [Coursework]

#### Learning objectives:

- To investigate alternative methods of determining calibrated frequency response transfer functions of a mechanical vibrating system, using a digital measuring system;
- To predict the response of a system from measured responses of its decoupled subsystems, and to compare with the measured response of the coupled system.

# **Practical information:**

Sessions will take place in the South Wing Mechanics Laboratory, throughout Lent term.

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• This activity doesn't involve preliminary work.

### Full Technical Report:

Students will have the option to submit a Full Technical Report.

# **Booklists**

Please see the **Booklist for Part IIA Courses** [2] for references for this module.

### **Examination Guidelines**

Please refer to Form & conduct of the examinations [3].

#### **UK-SPEC**

This syllabus contributes to the following areas of the **UK-SPEC** [4] standard:

Toggle display of UK-SPEC areas.

### GT1

Develop transferable skills that will be of value in a wide range of situations. These are exemplified by the Qualifications and Curriculum Authority Higher Level Key Skills and include problem solving, communication, and working with others, as well as the effective use of general IT facilities and information retrieval skills. They also include planning self-learning and improving performance, as the foundation for lifelong learning/CPD.

#### IA1

Apply appropriate quantitative science and engineering tools to the analysis of problems.

### KU1

Demonstrate knowledge and understanding of essential facts, concepts, theories and principles of their engineering discipline, and its underpinning science and mathematics.

# KU2

Have an appreciation of the wider multidisciplinary engineering context and its underlying principles.

### **E**1

Ability to use fundamental knowledge to investigate new and emerging technologies.

### **E2**

Ability to extract data pertinent to an unfamiliar problem, and apply its solution using computer based engineering tools when appropriate.

#### **E**3

Ability to apply mathematical and computer based models for solving problems in engineering, and the ability to assess the limitations of particular cases.

#### E4

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Understanding of and ability to apply a systems approach to engineering problems.

### **P1**

A thorough understanding of current practice and its limitations and some appreciation of likely new developments.

# US1

A comprehensive understanding of the scientific principles of own specialisation and related disciplines.

### US2

A comprehensive knowledge and understanding of mathematical and computer models relevant to the engineering discipline, and an appreciation of their limitations.

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#### Links

- [1] mailto:tb267@cam.ac.uk
- [2] https://www.vle.cam.ac.uk/mod/book/view.php?id=364091&chapterid=46411
- [3] https://teaching19-20.eng.cam.ac.uk/content/form-conduct-examinations
- [4] https://teaching19-20.eng.cam.ac.uk/content/uk-spec