# Engineering Tripos Part IB, 2P7: Vector Calculus, 2019-20

## Lecturer

Dr G Pullan [1]

## **Timing and Structure**

Weeks 1-3 and 6-8 Michaelmas term, 2 lectures/week; weeks 4-5 Michaelmas term, 1 lecture/week. 14 lectures

# Aims

The aims of the course are to:

• Provide the necessary background mathematics to ensure that students are confident in handling partial differential equations in vector form while maintaining a tangible physical appreciation of the manipulations involved.

# Objectives

As specific objectives, by the end of the course students should be able to:

- Differentiate and integrate scalar functions of two or more variables including transformations to other coordinate systems.
- Manipulate vector differential equations including the gradient, divergence and curl operators while retaining a physical appreciation of the mathematical operations involved.
- Perform line, surface and volume integrals and understand their various physical interpretations.
- Set up conservation statements in both differential and integral form and be able to transform from one to the other using Gauss's theorem.
- Appreciate the physical significance of curl and its relationship to circulation via Stokes's theorem in simple examples.
- Solve common PDE's (particularly the Laplace, Poisson, heat conduction and wave equations) with simple boundary conditions by the method of separation of variables.

# Content

The course provides an elementary introduction to vector calculus and aims to familiarise the student with the basic ideas of the differential calculus (the vector gradient, divergence and curl) and the integral calculus (line, surface and volume integrals and the theorems of Gauss and Stokes). The physical interpretation of the mathematical ideas will be stressed throughout via applications which centre on the derivation and manipulation of the common partial differential equations of engineering. The analytical solution of simple partial differential equations by the method of separation of variables will also be discussed.

A knowledge of the following Part IA lecture material on functions of more than one variable will be assumed: representation of curves and surfaces (including parametric representation); partial differentiation; total and perfect differentials; Taylor series; maxima and minima.

## The course will then consist of lectures on the following topics:

Vector functions and fields; field lines.

Vector differentiation; differentiation formulae.

The vector gradient and its physical interpretation;

Cylindrical and spherical polar co-ordinate systems.

The divergence and its physical interpretation; solenoidal fields; conservation statements;

Surface integrals; volume integrals; Gauss's divergence theorem; integral-differential transformations. Stokes's theorem.

The curl and its physical interpretation; irrotational fields; scalar potential; line integrals; conservative fields.

Types of PDE and boundary conditions; solution by separation of variables; examples of some common PDE's (Laplace, Poisson, heat conduction, wave equation

# **Booklists**

Please see the **Booklist for Part IB Courses** [2] for references for this module.

## **Examination Guidelines**

Please refer to Form & conduct of the examinations [3].

## **UK-SPEC**

This syllabus contributes to the following areas of the <u>UK-SPEC</u> [4] standard:

Toggle display of UK-SPEC areas.

#### IA1

Apply appropriate quantitative science and engineering tools to the analysis of problems.

#### KU1

Demonstrate knowledge and understanding of essential facts, concepts, theories and principles of their engineering discipline, and its underpinning science and mathematics.

#### KU2

Have an appreciation of the wider multidisciplinary engineering context and its underlying principles.

#### E2

Ability to extract data pertinent to an unfamiliar problem, and apply its solution using computer based engineering tools when appropriate.

## E3

Ability to apply mathematical and computer based models for solving problems in engineering, and the ability to assess the limitations of particular cases.

#### **P8**

Ability to apply engineering techniques taking account of a range of commercial and industrial constraints.

### US1

A comprehensive understanding of the scientific principles of own specialisation and related disciplines.

### US2

A comprehensive knowledge and understanding of mathematical and computer models relevant to the engineering discipline, and an appreciation of their limitations.

### US3

An understanding of concepts from a range of areas including some outside engineering, and the ability to apply them effectively in engineering projects.

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#### Links

[1] mailto:gp10006

[2] https://www.vle.cam.ac.uk/mod/book/view.php?id=364081&chapterid=43841

[3] https://teaching19-20.eng.cam.ac.uk/content/form-conduct-examinations

[4] https://teaching19-20.eng.cam.ac.uk/content/uk-spec